augmentation in the exterior radius r_0 (increasing the thickness of insulation) results in a reduction in the heat transfer rate in that segment of the cylinder. In the region where r_c is larger than r_0, an augmentation in thickness yields an increase in heat transfer rate there. Similar results are shown in Fig. 3 in which surface radiation is accounted for. With surface radiation the critical radius is expressed as $k/(h + 4\varepsilon F\sigma T_0^3)$ where in the present analysis ε and F are taken as unity. In this case the critical radius follows a similar trend to that in Fig. 2 but consistently lower. For horizontal cylinders, the present results are in a slight disagreement with the results of ref. [1] due to the difference in the relationships used for the natural convection heat transfer coefficient.

Figure 4 shows the effect of thickness and opacity of a semi-transparent insulation on the heat transfer rate from a horizontal cylinder. A horizontal case is chosen in order to isolate these effects from effects of inclination. The high rate of heat transfer exhibited for a zero absorption coefficient, $\kappa = 0$, is due to an additional factor, namely radiation leaving the inner surface of insulation at r_i with T_i directly to the exterior ambient in addition to the interior conduction and exterior convection. Increasing the opacity (increasing κ and r_0) shifts the rate of heat transfer towards the opaque case, and insulation in this case plays the role of a radiation shield. As the optical thickness ($r_0\kappa$) becomes very large, the insulation becomes essentially opaque and radiation becomes a surface phenomenon. This behavior can easily be verified by looking at the expression for the radiative heat flux $q_r)_{r_0^-}$ in the optically thick limit with black boundaries and temperature jump boundary conditions given as [6]

$$[q_r]_{r=r_0^-} = \frac{2\pi r_i \sigma[T_i^4 - T_0^4]}{\frac{3}{8}\left[2a_r r_i \ln r_0/r_i + \frac{1-(r_i/r_0)^2}{2a_r r_i}\right] + \frac{1}{2} + r_i/r_0}. \tag{10}$$

The value of this expression progressively decreases as a_r increases, and it approaches zero as a_r approaches infinity, thus yielding equation (6). Of interest is the location of the maximum value of heat transfer of the curves as they represent the position of the critical radius. The opaque insulation with surface radiation exhibits the lowest value for r_c. A reduction in the opacity shifts the peaks to the right and hence to a larger value for r_c. This shift can be interpreted as a result of providing an effective K, comprising radiation and conduction, and hence decreasing the value of the internal resistance to heat transfer. Finally, it can be stressed that in insulating slender cylinders arbitrarily oriented, a need arises in having the thickness of insulation compatable with the orientation as well as with the opacity of insulation. It is important to point out also that the idea of a local critical radius can be extended to a circumferentially variable heat transfer coefficient for the case of a horizontal cylinder. Extreme caution, however, should be exercised if the convection coefficient is considered to vary both axially and circumferentially over the cylinder due to the scatter of the existing data and the disagreement among the relationships recommended in the literature for natural convection from inclined surfaces.

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PERFORMANCE OF COUNTER-CURRENT HEAT EXCHANGER WITH PERIODIC INLET TEMPERATURES

M. S. Sodha

Centre of Energy Studies, Indian Institute of Technology, New Delhi 110016, India

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NOMENCLATURE

A_1, A_2,	areas of cross section of fluid channels;
c,	mean specific heat of the heat exchanger mass;
c_1, c_2,	specific heats of fluids at constant pressure;
$C_0(\mu_0 - 1)$, $C_0'(\mu_0 - 1)$,	constants of integration;
$C_{1m}(\mu_0 - 1)$, $C_{2m}(\mu_0 - 1)$,	constants of integration;
h,	heat transfer between the fluids per unit time per unit length per unit difference of temperature;
m,	integer $(1 - \infty)$;
M_1, M_2,	heat exchanger mass per unit length, corresponding to the two channels;
t,	time;
u_1,	$h/(A_1\rho_1c_1 + M_1c)$;
u_2,	$h/(A_2\rho_2c_2 + M_2c)$;
v_1,	$\left[\frac{A_1\rho_1c_1v_1'}{A_1\rho_1c_1 + M_1c}\right]$;
v_2,	$\left[\frac{A_2\rho_2c_2v_2'}{A_2\rho_2c_2 + M_2c}\right]$;
$v_1' - v_2'$,	fluid flow velocities;
x,	distance along heat exchanger, $(u_2/v_2) - (u_1/v_1)$.

Greek symbols

α_m^+, α_m^-,	complex roots of equation (8);
β_m,	$[(u_2 + im\omega)/v_2 + (\alpha_m u_1/v_1)]$;
μ_0,	$= (u_2/v_2)/(u_1/v_1)$;
ρ_1, ρ_2,	density of fluids;
ω,	2π/period.

INTRODUCTION

COUNTER current heat exchangers are commonly employed in solar hot water systems; the usual time independent analysis is not applicable because the inlet temperatures of hot water from the collector and cold water from the water supply are in general time dependent. This note presents an analysis of the heat exchanger when the inlet temperatures are periodic in nature.

The basic equations determining the performance of a counter current heat exchanger are

$$(M_1c + A_1\rho_1c_1)\frac{\partial T_1}{\partial t} + A_1\rho_1c_1v_1'\frac{\partial T_1}{\partial x} = h(T_2 - T_1) \quad (1a)$$

and

$$(M_2c + A_2\rho_2c_2)\frac{\partial T_2}{\partial t} - A_2\rho_2c_2v_2'\frac{\partial T_2}{\partial x} = h(T_1 - T_2) \quad (1b)$$

where M_1 and M_2 are the masses per unit length of the heat exchanger, associated with the two channels of the heat exchanger. Since there is a temperature distribution within the mass of the heat exchanger, M_1 and M_2 cannot be assigned *a priori* values; M_1 and M_2 have to be chosen for the best data fit so that $M_1 + M_2$ is equal to the total mass per unit length of the heat exchanger.

The periodic boundary conditions are

$$T_1 = a_{10} + \sum_{m=1}^{\infty} a_{1m}\exp(im\omega t) \quad \text{at } x = 0 \quad (2a)$$

and

$$T_2 = a_{20} + \sum_{m=1}^{\infty} a_{2m}\exp(im\omega t) \quad \text{at } x = L. \quad (2b)$$

In general the overall heat transfer coefficient between the fluids, h, is a function of T_1 and T_2; we have however assumed it to be constant, implying small changes in T_1 and T_2.

The steady state periodic behaviour of T_1 and T_2 may be expressed as

$$T_1 = T_{10}(x) + \sum_{m=1}^{\infty} T_{1m}(x)\exp(im\omega t) \quad (3a)$$

and

$$T_2 = T_{20}(x) + \sum_{m=1}^{\infty} T_{2m}(x)\exp(im\omega t). \quad (3b)$$

Substituting for T_1 and T_2 from equation (3) in equation (1) one obtains

$$\frac{dT_{10}}{dx} = \frac{u_1}{v_1}(T_{20} - T_{10}), \quad (4a)$$

$$\frac{dT_{20}}{dx} = \frac{u_2}{v_2}(T_{20} - T_{10}) \quad (4b)$$

$$\frac{dT_{1m}}{dx} = \frac{u_1}{v_1}T_{2m} - \frac{(u_1 + im\omega)}{v_1}T_{1m} \quad (5a)$$

and

$$\frac{dT_{2m}}{dx} = \frac{u_2 + im\omega}{v_2}T_{2m} - \frac{u_2}{v_2}T_{1m}. \quad (5b)$$

From equations (4a) and (4b) one obtains

$$\frac{d(T_{20} - T_{10})}{dx} = \left(\frac{u_2}{v_2} - \frac{u_1}{v_1}\right)(T_{20} - T_{10})$$

and

$$\frac{dT_{20}}{dx} = \frac{u_2}{v_2}\frac{v_1}{u_1}\frac{dT_{10}}{dx}.$$

Integrating the above equations one obtains

$$T_{20} - T_{10} = C_0\exp(\alpha_0 x)\cdot(\mu_0 - 1)$$

and

$$T_{20} = \mu_0 T_{10} + C_0'(\mu_0 - 1).$$

Hence using equation (2a) one obtains

$$T_{10} = C_0\exp(\alpha_0 x) - C_0' \quad (6a)$$

and

$$T_{20} = \mu_0 C_0\exp(\alpha_0 x) - C_0' \quad (6b)$$

where

$$C_0 = (a_{20} - a_{10})/[\mu_0\exp(\alpha_0 L) - 1] \quad (6c)$$

and

$$C_0' = [a_{20} - a_{10}\mu_0\exp(\alpha_0 L)]/[\mu_0\exp(\alpha_0 L) - 1]. \quad (6d)$$

From equations (5a) and (5b) one obtains

$$\begin{aligned}\frac{d}{dx}(T_{2m} + \alpha_m T_{1m}) &= \left(\frac{u_2 + im\omega}{v_2} + \alpha_m\frac{u_1}{v_1}\right)T_{2m} \\ &\quad - \left(\frac{u_2}{v_2} + \alpha_m\frac{(u_1 + im\omega)}{v_1}\right)T_{1m} \\ &= \beta_m(T_{2m} + \alpha_m T_{1m}) \end{aligned} \quad (7)$$

where

$$\alpha_m = -\left[\frac{\dfrac{u_2}{v_2} + \dfrac{\alpha_m}{v_1}(u_1 + im\omega)}{\dfrac{u_2 + im\omega}{v_2} + \alpha_m\dfrac{u_1}{v_1}}\right]. \quad (8)$$

Designating the two roots of the quadratic equation (8) by α_m^+ and α_m^- and the corresponding values of β_m by β_m^+ and β_m^- and integrating equation (7) one obtains

$$T_{2m} + \alpha_m^+ T_{1m} = C_m^+\exp(\beta_m^+ x)(\alpha_m^+ - \alpha_m^-)$$

and

$$T_{2m} + \alpha_m^- T_{1m} = C_m^-\exp(\beta_m^- x)(\alpha_m^+ - \alpha_m^-).$$

From the above set of equations one obtains

$$T_{1m} = C_m^+\exp(\beta_m^+ x) - C_m^-\exp(\beta_m^- x) \quad (9a)$$

and

$$T_{2m} = \alpha_m^+ C_m^-\exp(\beta_m^- x) - \alpha_m^- C_m^+\exp(\beta_m^+ x). \quad (9b)$$

Using the boundary condition given by equation (3)

$$C_m^+ = [a_{1m}\alpha_m^+\exp(\beta_m^- L) + a_{2m}]/[\alpha_m^+\exp(\beta_m^- L) - \alpha_m^-\exp(\beta_m^+ L)] \quad (9c)$$

$$C_m^- = [a_{1m}\alpha_m^-\exp(\beta_m^+ L) + a_{2m}]/[\alpha_m^+\exp(\beta_m^- L) - \alpha_m^-\exp(\beta_m^+ L)] \quad (9d)$$

Thus the time and space dependence of the given temperature of the fluid is given by equation (2) with T_{10}, T_{20} given by equation (6) and T_{1m}, T_{2m} given by equation (9). The outlet temperatures are obtained by putting $x = L$ in equation (3a) and $x = 0$ in equation (3b). It may be remembered that $b_1 + ib_2 = A\exp(i\phi)$ when b_1 is positive and $b_1 + ib_2 = A\exp[i(\pi + \phi)]$ when b_1 is negative, where $A = |(A^2 + B^2)^{1/2}|$, $\phi = \tan^{-1}(b_2/b_1)$ and $|\phi| < \pi/2$; failure to keep to this causes mistakes.